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Nontrivial Correlation between the CKM and MNS Matrices

Zhi-zhong Xing
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
and Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918 (4), Beijing 100039, China *
(Electronic address: xingzz@mail.ihep.ac.cn)

Abstract

We point out that the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V_{\rm CKM}$ and the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix $V_{\rm MNS}$ can naturally be correlated in a class of seesaw models with grand unification, but the texture of their correlation matrix $\mathcal{F}_\nu$ is rather nontrivial. The bimaximal mixing pattern of $\mathcal{F}_\nu$ is disfavored by current data, and other special forms of $\mathcal{F}_\nu$ may suffer from fine-tuning of the free phase parameters in fitting the so-called quark-lepton complementarity relation. A straightforward calculation of $\mathcal{F}_\nu$ in terms of $V_{\rm CKM}$ and $V_{\rm MNS}$ reveals a striking feature of $\mathcal{F}_\nu$: its (1,3) element cannot be zero or too small, no matter whether the (1,3) elements of $V_{\rm CKM}$ and $V_{\rm MNS}$ are vanishing or not. We also add some brief comments on possible radiative corrections to $V_{\rm CKM}$ and $V_{\rm MNS}$.

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*Mailing address

$\boxed{1}$ Recent solar [1], atmospheric [2], reactor (KamLAND [3] and CHOOZ [4]) and accelerator (K2K [5]) neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive and lepton flavors are mixed. The phenomenon of lepton flavor mixing can be described by a 3×3 unitary matrix $V_{\rm MNS}$, commonly referred to as the Maki-Nakagawa-Sakata (MNS) matrix [6]. Current experimental data indicate that $V_{\rm MNS}$ involves two large angles ($\theta_{12} \sim 33^\circ$ and $\theta_{23} \sim 45^\circ$) and one small angle ($\theta_{13} \lesssim 9^\circ$) in the standard parametrization. The magnitude of θ_{13} remains unknown, but a global analysis of the presently available neutrino oscillation data [7] hints that $\theta_{13} \sim 3^\circ$ seems to be most likely. On the other hand, three nontrivial CP-violating phases of $V_{\rm MNS}$ (denoted by $\delta_{\rm MNS}$, ρ and σ) are entirely unrestricted. One particularly important target of the future neutrino experiments is just to measure θ_{13} and $\delta_{\rm MNS}$ (of Dirac type) and to constrain ρ and σ (of Majorana type).

In comparison with the MNS matrix $V_{\rm MNS}$, the Cabibbo-Kobayashi-Maskawa (CKM) quark flavor mixing matrix $V_{\rm CKM}$ involves three small mixing angles and one large CP-violating phase ($\vartheta_{12} \approx 13^\circ$, $\vartheta_{23} \approx 2.4^\circ$, $\vartheta_{13} \approx 0.2^\circ$ and $\delta_{\rm CKM} \sim 65^\circ$ in the standard parametrization [8]). The apparent difference between the CKM and MNS matrices requires a good dynamical reason in a fundamental theory of flavor mixing and CP violation, in particular when an underlying lepton-quark symmetry is concerned. Some phenomenological speculation about possible relations between θ_{ij} and ϑ_{ij}, such as [9–11]

$$\theta_{12} + \vartheta_{12} \; \approx \; 45^\circ \tag{1}$$

and [9] [1]

$$\theta_{23} + \vartheta_{23} \; \approx \; 45^\circ \; , \tag{2}$$

has appeared [2]. If such relations could survive the test with more accurate experimental data in the near future, would they be just accidental or imply a kind of lepton-quark symmetry? The latter is certainly attractive to theorists, although it remains unclear what symmetry exists between leptons and quarks.

The main purpose of this paper is to investigate a simple but natural relation between $V_{\rm MNS}$ and $V_{\rm CKM}$ for a large class of seesaw models with grand unification,

$$V_{\rm MNS} \; = \; V^\dagger_{\rm CKM} {\cal F}_\nu \; , \tag{3}$$

in which ${\cal F}_\nu$ denotes a unitary matrix associated with the diagonalization of the effective neutrino mass matrix M_ν [12]. We shall show that it is phenomenologically disfavored for ${\cal F}_\nu$ to take the bimaximal mixing form, even if the nontrivial phase effects in ${\cal F}_\nu$ are taken into account. This observation is contrary to some previous arguments that the CKM matrix might just measure the deviation of the MNS matrix from exact bimaximal mixing [9,10,13].

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[1]Note that $\theta_{23} - \vartheta_{23} \approx 45^\circ$ can indistinguishably be expected from current experimental data.

[2]Note that $\theta_{13} \sim \vartheta_{13}$ was also mentioned in Ref. [9]. However, we find that the present best-fit value of θ_{13} favors $\theta_{13} \sim \vartheta_{23}$ instead of $\theta_{13} \sim \vartheta_{13}$.

Furthermore, we point out that a slight modification of the bimaximal mixing pattern of $\mathcal{F}_\nu$ will allow us to reproduce the so-called quark-lepton complementarity relation in Eq. (1), but the fine-tuning of relevant unknown parameters seems unavoidable. A straightforward calculation of $\mathcal{F}_\nu$ in terms of the mixing angles and CP-violating phases of $V_{\rm CKM}$ and $V_{\rm MNS}$ reveals a striking feature of $\mathcal{F}_\nu$: its (1,3) element cannot be vanishing or too small, no matter whether θ_{13} and ϑ_{13} are taken to be zero or not. Therefore we conclude that the texture of $\mathcal{F}_\nu$ is rather nontrivial in the seesaw models. We also make some brief comments on possible quantum corrections to fermion mass matrices and their threshold effects on the flavor mixing parameters that are related to one another by Eq. (3).

$\boxed{\mathbf{2}}$ First of all, let us explain why Eq. (3) naturally holds for a class of seesaw models with lepton-quark unification. It is well-known that the CKM matrix $V_{\rm CKM} \equiv V^\dagger_{\rm u} V_{\rm d}$ arises from the mismatch between the diagonalization of the up-type quark mass matrix $M_{\rm u}$ and that of the down-type quark mass matrix $M_{\rm d}$:

$$\begin{aligned} M_{\rm u} & = V_{\rm u} \, \overline{M}_{\rm u} \, U^\dagger_{\rm u} \; , \\ M_{\rm d} & = V_{\rm d} \, \overline{M}_{\rm d} \, U^\dagger_{\rm d} \; , \end{aligned} \tag{4}$$

where $\overline{M}_{\rm u} = {\rm Diag}\{m_u, m_c, m_t\}$, $\overline{M}_{\rm d} = {\rm Diag}\{m_d, m_s, m_b\}$, $U_{\rm u,d}$ and $V_{\rm u,d}$ are unitary matrices. Similarly, the MNS matrix $V_{\rm MNS} \equiv V^\dagger_l V_\nu$ comes from the mismatch between the diagonalization of the charged lepton mass matrix M_l and that of the (effective) Majorana neutrino mass matric M_ν:

$$\begin{aligned} M_l & = V_l \, \overline{M}_l \, U^\dagger_l \; , \\ M_\nu & = V_\nu \, \overline{M}_\nu \, V^T_\nu \; , \end{aligned} \tag{5}$$

in which $\overline{M}_l = {\rm Diag}\{m_e, m_\mu, m_\tau\}$, $\overline{M}_\nu = {\rm Diag}\{m_1, m_2, m_3\}$, $V_{l,\nu}$ and U_l are unitary matrices. Taking account of the canonical (Type I) seesaw mechanism [14], we have

$$M_\nu = M_{\rm D} M^{-1}_{\rm R} M^T_{\rm D} \; , \tag{6}$$

where $M_{\rm D}$ and $M_{\rm R}$ stand respectively for the Dirac neutrino mass matrix and the heavy (right-handed) Majorana neutrino mass matrix. A diagonalization of $M_{\rm D}$ is straightforward:

$$M_{\rm D} = V_{\rm D} \, \overline{M}_{\rm D} \, U^\dagger_{\rm D} \; , \tag{7}$$

where $\overline{M}_{\rm D} = {\rm Diag}\{m_x, m_y, m_z\}$ with m_x, m_y and m_z being the (positive) eigenvalues of $M_{\rm D}$, $V_{\rm D}$ and $U_{\rm D}$ are unitary matrices. Since the idea of SO(10) grand unification [15] provides a natural framework in which leptons and quarks are correlated and the seesaw mechanism automatically works [3], we just concentrate on it in the following. But we restrict ourselves to a phenomenologically simplified and interesting scenario: $M_{\rm D} = M_{\rm u}$ and $M_l = M_{\rm d}$, and all of

[3]It is worth pointing out that the non-canonical (Type II) seesaw mechanism works more naturally in most of the realistic SO(10) models [16]. For simplicity, here we assume that the canonical seesaw relation in Eq. (6) holds as a leading-order approximation of the non-canonical seesaw relation.

them are symmetric [17]. We are then left with $V_{\rm D} = U_{\rm D}^* = V_{\rm u} = U_{\rm u}^*$ and $V_l = U_l^* = V_{\rm d} = U_{\rm d}^*$, as well as $m_x = m_u$, $m_y = m_c$ and $m_z = m_t$. A relation between the MNS matrix $V_{\rm MNS}$ and the CKM matrix $V_{\rm CKM}$ turns out to be

$$V_{\rm MNS} \; = \; V_{\rm d}^\dagger V_\nu \; = \; V_{\rm CKM}^\dagger \left(V_{\rm u}^\dagger V_\nu \right) \; = \; V_{\rm CKM}^\dagger \mathcal{F}_\nu \; , \tag{8}$$

where $\mathcal{F}_\nu \equiv V_{\rm u}^\dagger V_\nu$. With the help of the seesaw relation in Eq. (6), the role of $\mathcal{F}_\nu$ can be seen clearly:

$$\mathcal{F}_\nu \overline{M}_\nu \mathcal{F}_\nu^T \; = \; \overline{M}_{\rm u} V_{\rm u}^T M_{\rm R}^{-1} V_{\rm u} \overline{M}_{\rm u} \; . \tag{9}$$

In other words, the unitary matrix $\mathcal{F}_\nu$ transforms the combination $\overline{M}_{\rm u} V_{\rm u}^T M_{\rm R}^{-1} V_{\rm u} \overline{M}_{\rm u}$ into the diagonal (physical) mass matrix of three light neutrinos. Given $\mathcal{F}_\nu$ and $V_{\rm u}$, the heavy Majorana neutrino mass matrix $M_{\rm R}$ is determined by

$$M_{\rm R} \; = \; V_{\rm u} \overline{M}_{\rm u} \mathcal{F}_\nu^* \overline{M}_\nu^{-1} \mathcal{F}_\nu^\dagger \overline{M}_{\rm u} V_{\rm u}^T \; . \tag{10}$$

This *inverted* seesaw relation indicates that the mass scale of three right-handed neutrinos is roughly m_t^2/m_3, if the light neutrino mass spectrum is essentially hierarchical.

We argue that Eq. (8) is a natural relation between CKM and MNS matrices in a class of seesaw models with lepton-quark unification. It will not be favored by current neutrino oscillation data, however, if $\mathcal{F}_\nu$ takes the bimaximal mixing pattern.

$\boxed{\mathbf{3}}$ Because $\mathcal{F}_\nu$ depends on both $M_{\rm R}$ and $V_{\rm u}$ through Eq. (9), it is in general a complex unitary matrix. The most generic form of $\mathcal{F}_\nu$ with bimaximal mixing can be written as

$$\mathcal{F}_\nu \; = \; P_\nu \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} Q_\nu \; , \tag{11}$$

where $P_\nu \equiv {\rm Diag}\{e^{i\phi_x}, e^{i\phi_y}, e^{i\phi_z}\}$ and $Q_\nu \equiv {\rm Diag}\{e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\}$ are two phase matrices. Taking account of Eq. (8), one may choose two independent phases (or their combinations) of Q_ν as the Majorana-type CP-violating phases of $V_{\rm MNS}$. On the other hand, the phases of P_ν can affect both the mixing angles and the Dirac-type CP-violating phase of $V_{\rm MNS}$. To see this point in a more transparent way, we make use of the standard parametrizations of $V_{\rm CKM}$ and $V_{\rm MNS}$ [18],

$$\begin{aligned} V_{\rm CKM} &= R_{23}(\vartheta_{23}) \otimes \Gamma_\delta(\delta_{\rm CKM}) \otimes R_{13}(\vartheta_{13}) \otimes R_{12}(\vartheta_{12}) \; , \\ V_{\rm MNS} &= R_{23}(\theta_{23}) \otimes \Gamma_\delta(\delta_{\rm MNS}) \otimes R_{13}(\theta_{13}) \otimes R_{12}(\theta_{12}) \otimes Q_\nu \; , \end{aligned} \tag{12}$$

in which R_{ij} denotes the rotation matrix in the (i,j)-plane with the mixing angle θ_{ij} or ϑ_{ij} (for $ij = 12, 23, 13$), and $\Gamma_\delta(\delta) \equiv {\rm Diag}\{1, 1, e^{i\delta}\}$ is a phase matrix consisting of the Dirac phase of CP violation. When Eq. (12) is applied to Eq. (8), a proper rephasing of three charged lepton fields has been implied for the sake of phase consistency on the two sides of

Eq. (8) [4]. We are then allowed to calculate three mixing angles of $V_{\rm MNS}$ in terms of those of $V_{\rm CKM}$. The approximate results are [5]

$$\begin{aligned}
\tan\theta_{12} &\approx \frac{\sqrt{1-\sqrt{2}\tan\vartheta_{12}\cos(\phi_y-\phi_x)}}{\sqrt{1+\sqrt{2}\tan\vartheta_{12}\cos(\phi_y-\phi_x)}} \; , \\
\tan\theta_{23} &\approx \cos\vartheta_{12} \; , \\
\sin\theta_{13} &\approx \frac{1}{\sqrt{2}}\sin\vartheta_{12} \; ,
\end{aligned} \tag{13}$$

where the strong hierarchy of three quark mixing angles has been taken into account. In addition, the Jarlskog invariant of CP violation [19] defined for $V_{\rm MNS}$ is given by

$$\mathcal{J}_{\rm MNS} \;\approx\; \frac{1}{8\sqrt{2}}\sin 2\vartheta_{12}\sin(\phi_y-\phi_x) \tag{14}$$

to the leading order, only if the phase difference $(\phi_y-\phi_x)$ is not too close to 0 or $\pm\pi$. Then the maximally-allowed magnitude of $\mathcal{J}_{\rm MNS}$ is $|\mathcal{J}_{\rm MNS}| \approx 4\%$ for $(\phi_y-\phi_x)\approx \pm\pi/2$. But this special phase condition is not favored by Eq. (13), because it will lead to $\tan\theta_{12}\approx 1$ or $\theta_{12}\approx 45^\circ$. Taking account of $\mathcal{J}_{\rm MNS} = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta_{\rm MNS}$, we are then able to determine the Dirac phase $\delta_{\rm MNS}$ from Eqs. (13) and (14). We obtain $\delta_{\rm MNS}\approx(\phi_y-\phi_x)$ in the leading-order approximation. Therefore the phase matrix P_ν of $\mathcal{F}_\nu$, which may significantly affect θ_{12} and $\delta_{\rm MNS}$ of $V_{\rm MNS}$, should not be ignored.

Two more comments on the consequences of Eqs. (13) and (14) are in order.

(a) At first glance, it is absolutely impossible to reproduce the empirical relations given in Eqs. (1) and (2) from Eq. (13), simply because of the existence of unknown (ϕ_x, ϕ_y, ϕ_z) phases. It is worth emphasizing that the special assumption $\phi_x=\phi_y=\phi_z=0$ taken in the literature is not justified, since $\mathcal{F}_\nu$ is naturally expected to be complex in the seesaw models. For this reason, we argue that Eqs. (1) and (2) are most likely to be a numerical accident.

(b) Even if $\phi_x=\phi_y=\phi_z=0$ is assumed, it will be difficult to straightforwardly derive Eq. (1) from Eq. (13). In this case, the expression of $\tan\theta_{12}$ in Eq. (13) is simplified to

$$\tan\theta_{12} \;\approx\; \frac{\sqrt{1-\sqrt{2}\tan\vartheta_{12}}}{\sqrt{1+\sqrt{2}\tan\vartheta_{12}}} \; , \tag{15}$$

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[4]A general parametrization of $V_{\rm CKM}$ should include two (unobservable) phase matrices on its left-hand side (denoted as P) and its right-hand side (denoted as Q). Taking account of Eq. (8), we find that $P^\dagger$ can be absorbed into P_ν of $\mathcal{F}_\nu$, while $Q^\dagger$ can be removed by a redefinition of the non-physical phases of three charged lepton fields. Therefore, the standard parametrizations of $V_{\rm CKM}$ and $V_{\rm MNS}$ given in Eq. (12) together with the most generic form of $\mathcal{F}_\nu$ taken in Eq. (11) are consistent with Eq. (8) and the Lagrangian of lepton and quark Yukawa interactions.

[5]To leading order in our approximation, another (unobservable) phase combination $(\phi_z-\phi_y)$ does not appear in Eqs. (13) and (14).

Typically taking $\vartheta_{12} \approx 13°$, we obtain $\theta_{12} \approx 35°$ from Eq. (15). In addition, $\theta_{23} \approx 44°$ and $\theta_{13} \approx 9°$ can be obtained from Eq. (13). It turns out that the sum of θ_{12} and ϑ_{12} amounts to 48°, which is not perfectly in agreement with Eq. (1). On the other hand, the prediction $\theta_{13} \approx 9°$ is too close to the present experimental upper limit and far away from the best-fit result (namely, $\theta_{13} \sim 3°$ [7]). Note also that the assumption $\phi_x = \phi_y = \phi_z = 0$ leads to tiny CP violation in the lepton sector:

$$|\mathcal{J}_{\rm MNS}| \;\approx\; \frac{1}{4\sqrt{2}} \sin\vartheta_{13} \sin\delta_{\rm CKM} \; , \tag{16}$$

where only the leading term has been given. Numerically, $|\mathcal{J}_{\rm MNS}| \sim 5 \times 10^{-4}$, too small to be measured in the future long-baseline neutrino oscillation experiments [20].

In order to successfully achieve $\theta_{12} + \vartheta_{12} \approx 45°$ from $V_{\rm MNS} = V^\dagger_{\rm CKM} \mathcal{F}_\nu$, a straightforward (and somehow trivial) way is to modify the bimaximal mixing pattern of $\mathcal{F}_\nu$. For illustration, let us consider the following form of $\mathcal{F}_\nu$:

$$\mathcal{F}_\nu = P_\nu \left(\begin{matrix} c & s & 0 \\ -\frac{1}{\sqrt{2}} s & \frac{1}{\sqrt{2}} c & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} s & -\frac{1}{\sqrt{2}} c & \frac{1}{\sqrt{2}} \end{matrix} \right) Q_\nu \; , \tag{17}$$

where $c \equiv \cos\theta$ and $s \equiv \sin\theta$ with θ being an unspecified angle. Eq. (11) can obviously be reproduced from Eq. (17) by taking $\theta = 45°$. Taking account of the correlation between $V_{\rm MNS}$ and $V_{\rm CKM}$ in Eq. (3) or (8), we now arrive at

$$\begin{aligned} \tan\theta_{12} &\approx \frac{\sqrt{\tan\theta - \sqrt{2}\tan\vartheta_{12}\cos(\phi_y - \phi_x)}}{\sqrt{\cot\theta + \sqrt{2}\tan\vartheta_{12}\cos(\phi_y - \phi_x)}} \\ \mathcal{J}_{\rm MNS} &\approx \frac{1}{8\sqrt{2}} \sin 2\theta \sin 2\vartheta_{12} \sin(\phi_y - \phi_x) \; , \end{aligned} \tag{18}$$

in the leading-order approximation. The results for $\tan\theta_{23}$ and $\sin\theta_{13}$ obtained in Eq. (13) keep unchanged. The relation $\theta_{12} + \vartheta_{12} \approx 45°$, if it holds, implies a kind of correlation between θ and $(\phi_y - \phi_x)$ as restricted by Eq. (18). To be more concrete, we find that this correlation reads as

$$\cos(\phi_y - \phi_x) \;\approx\; \frac{2\tan\vartheta_{12} - \cos 2\theta}{\sqrt{2}\tan\vartheta_{12}\sin 2\theta} \tag{19}$$

to the lowest order. Eq. (19) is numerically illustrated in Fig. 1(A), where $\vartheta_{12} \approx 13°$ has typically been taken. One can clearly see that the value of θ is sensitive to that of $(\phi_y - \phi_x)$ and the possibility of $\theta = 45°$ has been ruled out. If $\theta = 30°$ [21] or $\theta \approx 35.3°$ [22] is assumed, then the value of $(\phi_y - \phi_x)$ must be fine-tuned to about 100° or 70°. In these two cases, the magnitude of $\mathcal{J}_{\rm MNS}$ can be as large as about 3%, as shown in Fig. 1(B).

We conclude that current experimental data do not favor the bimaximal mixing pattern of $\mathcal{F}_\nu$ taken for Eq. (3) or (8), no matter whether $\mathcal{F}_\nu$ is real or complex. The numerical relation $\theta_{12} + \vartheta_{12} \approx 45°$ is achievable, however, if $\mathcal{F}_\nu$ is allowed to slightly deviate from the bimaximal mixing form and its free parameters can properly be fine-tuned. Of course, such a phenomenological approach is not well motivated from a theoretical point of view.

$\boxed{4}$ What is the most favorite pattern of $\mathcal{F}_\nu$ in phenomenology? One can straightforwardly answer this question by calculating $\mathcal{F}_\nu$ in terms of $V_{\rm CKM}$ and $V_{\rm MNS}$ through Eq. (3) or (8); namely, $\mathcal{F}_\nu = V_{\rm CKM} V_{\rm MNS}$. When the standard parametrization of $V_{\rm CKM}$ or $V_{\rm MNS}$ in Eq. (12) is used, one should take account of the arbitrary but nontrivial phases between $V_{\rm CKM}$ and $V_{\rm MNS}$. In this case [6],

$$\mathcal{F}_\nu = V_{\rm CKM}(\vartheta_{12}, \vartheta_{23}, \vartheta_{13}, \delta_{\rm CKM}) \otimes \Omega_\nu \otimes V_{\rm MNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\rm MNS}) \; , \tag{20}$$

where $\Omega_\nu \equiv {\rm Diag}\{e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}\}$ is the relative phase matrix between the CKM and MNS matrices. To be more specific, we fix $(\vartheta_{12}, \vartheta_{23}, \vartheta_{13}) \approx (13^\circ, 2.4^\circ, 0.2^\circ)$ [8] and $(\theta_{12}, \theta_{23}, \theta_{13}) \approx (33^\circ, 45^\circ, 3^\circ)$ in our numerical calculation of $\mathcal{F}_\nu$. It proves very instructive and convenient to expand the matrix elements of $V_{\rm CKM}$ and $V_{\rm MNS}$ in powers of a small parameter $\lambda \equiv \sin\vartheta_{12}$:

$$\begin{aligned} V_{\rm CKM}(\vartheta_{12}, \vartheta_{23}, \vartheta_{13}, \delta_{\rm CKM}) &\approx \begin{pmatrix} \lambda^{0.02} & \lambda^{1.00} & \lambda^{3.79} e^{-i\delta_{\rm CKM}} \\ -\lambda^{1.00} & \lambda^{0.02} & \lambda^{2.12} \\ \lambda^{3.13} - \lambda^{3.79} e^{i\delta_{\rm CKM}} & -\lambda^{2.12} & \lambda^{0.00} \end{pmatrix} \; , \\ V_{\rm MNS}(\theta_{12}, \; \theta_{23}, \; \theta_{13}, \; \delta_{\rm MNS}) &\approx \begin{pmatrix} \lambda^{0.12} & \lambda^{0.41} & \lambda^{1.98} e^{-i\delta_{\rm MNS}} \\ -\lambda^{0.64} & \lambda^{0.35} & \lambda^{0.23} \\ \lambda^{0.64} - \lambda^{2.33} e^{i\delta_{\rm MNS}} & -\lambda^{0.35} & \lambda^{0.23} \end{pmatrix} \; , \end{aligned} \tag{21}$$

in which only the leading term of each matrix element (except the (3,1) elements of $V_{\rm MNS}$) is shown. From Eqs. (20) and (21), we obtain

$$\mathcal{F}_\nu \approx \Omega_\nu \begin{pmatrix} \lambda^{0.14}(1 - \lambda^{1.50} e^{i\delta_{21}}) & \lambda^{0.43}(1 + \lambda^{0.92} e^{i\delta_{21}}) & \lambda^{1.23}(e^{i\delta_{21}} + \lambda^{0.77} e^{-i\delta_{\rm MNS}}) \\ -\lambda^{0.66}(1 + \lambda^{0.46} e^{-i\delta_{21}}) & \lambda^{0.37}(1 - \lambda^{1.04} e^{-i\delta_{21}}) & \lambda^{0.25}(1 + \lambda^{2.10} e^{i\delta_{32}}) \\ \lambda^{0.64}(1 - \lambda^{1.69} e^{i\delta_{\rm MNS}}) & -\lambda^{0.35}(1 + \lambda^{2.12} e^{-i\delta_{32}}) & \lambda^{0.23}(1 - \lambda^{2.12} e^{-i\delta_{32}}) \end{pmatrix} \; , \tag{22}$$

where $\delta_{ij} \equiv \delta_i - \delta_j$ (for $i, j = 1, 2, 3$) has been defined. One can see that the CP-violating phase $\delta_{\rm CKM}$ does not play a role in this approximation. The most striking feature of $\mathcal{F}_\nu$ is that its (1,3) element does not vanish, unlike the ansatz proposed in Eq. (11) or (17). This result remains valid even in the $\theta_{13} = 0$, $\vartheta_{13} = 0$ or $\theta_{13} = \vartheta_{13} = 0$ case, simply because $|(\mathcal{F}_\nu)_{13}|$ is dominated by the product $|(V_{\rm CKM})_{us} (V_{\rm MNS})_{\mu 3}| = \sin\vartheta_{12} \sin\theta_{23} \cos\vartheta_{13} \cos\theta_{13} \approx \lambda^{1.23}$, as already shown in Eq. (22). Allowing the unknown phase parameters $\delta_{\rm MNS}$, δ_{21} and δ_{32} to vary between 0 and 2π, we arrive at the possible ranges of nine matrix elements of $\mathcal{F}_\nu$:

$$|\mathcal{F}_\nu| \approx \begin{pmatrix} 0.72 \cdots 0.90 & 0.39 \cdots 0.66 & 0.11 \cdots 0.21 \\ 0.18 \cdots 0.56 & 0.45 \cdots 0.70 & 0.66 \cdots 0.72 \\ 0.35 \cdots 0.42 & 0.57 \cdots 0.62 & 0.68 \cdots 0.74 \end{pmatrix} \; . \tag{23}$$

Note here that the ranges given above are for the *individual* matrix elements. The choice of a specific value for one element may restrict the magnitudes of some others, because they are related to one another by the unitarity conditions of $\mathcal{F}_\nu$.

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[6]Here we have omitted the Majorana phases of CP violation in $V_{\rm MNS}$, as they can always be rearranged into a pure phase matrix on the right-hand side of $V_{\rm MNS}$, just like Q_ν in Eq. (11). This simplification does not affect our estimation of the moduli of nine matrix elements of $\mathcal{F}_\nu$.

Eqs. (22) and (23) illustrate that the correlation between $V_{\rm MNS}$ and $V_{\rm CKM}$ is rather nontrivial. In particular, the correlation matrix $\mathcal{F}_\nu$ is neither real nor bimaximal. Such a result is of course not a surprise, because Eq. (9) has clearly indicated that $\mathcal{F}_\nu$ should not have a too special pattern. Considering the fact that $\mathcal{F}_\nu$ consists of one small rotation angle in its (1,3) sector and two large rotation angles in its (1,2) and (2,3) sectors, we expect that the texture of $M_{\rm R}$ must be very nontrivial too.

In the above discussions, we did not take into account possible quantum corrections to relevant physical parameters between the scale of grand unified theories $\Lambda_{\rm GUT}$ and the low energy scales $\Lambda_{\rm LOW}$ at which the mixing angles and CP-violating phases of $V_{\rm CKM}$ and $V_{\rm MNS}$ can experimentally be determined. One may roughly classify such renormalization effects into three categories in the afore-mentioned seesaw models with grand unification:

- The first category is about radiative corrections to the fermion mass and flavor mixing parameters between the GUT scale $\Lambda_{\rm GUT}$ and the seesaw scale M_1, where M_1 denotes the mass of the lightest right-handed neutrino. Typically, $\Lambda_{\rm GUT} \sim 10^{15\cdots 16}$ GeV and $M_1 \sim 10^{8\cdots 12}$ GeV hold in most model-building cases [23]. Because three right-handed neutrinos are in general expected to have a mass hierarchy ($M_1 < M_2 < M_3$), the threshold effects in the renormalization chain $\Lambda_{\rm GUT} \rightarrow M_3 \rightarrow M_2 \rightarrow M_1$ are likely to modify the parameters of neutrino masses and lepton flavor mixing in a significant way [24]. The explicit estimation of such radiative corrections involves a number of free parameters [25], hence its arbitrariness or uncertainty is essentially out of control. In contrast, quark flavor mixing is expected to be insensitive to such seesaw threshold effects.

- From the seesaw scale M_1 to the electroweak scale $\Lambda_{\rm EW}$ ($\sim 10^2$ GeV), the one-loop renormalization group equation of the effective neutrino mass matrix M_ν consists of the contributions from gauge interactions, quark Yukawa interactions and charged-lepton Yukawa interactions [26]. Only the last contribution may affects $V_{\rm MNS}$, but the quantitative corrections to lepton mixing angles and CP-violating phases are strongly suppressed in the standard model and in the supersymmetric standard model with small $\tan\beta$ (for simplicity, we have assumed that the scale of supersymmetry breaking is not far away from $\Lambda_{\rm EW}$). The running behavior of $V_{\rm CKM}$ from M_1 to $\Lambda_{\rm EW}$ is in general dominated by the t-quark, b-quark and τ-lepton Yukawa couplings. To be explicit, ϑ_{23}, ϑ_{13} and $\delta_{\rm CKM}$ are sensitive to a drastic change of energy scales, but ϑ_{12} is not [27].

- Within the standard model, the $\Lambda_{\rm EW}$ threshold effect is negligibly small for both leptons and quarks. Thus it is unnecessary to take into account the radiative correction to $V_{\rm MNS}$ and $V_{\rm CKM}$ from $\Lambda_{\rm EW}$ to $\Lambda_{\rm LOW}$. However, the $\Lambda_{\rm EW}$ threshold effect may be important in the supersymmetric case [26]; e.g., it can be induced by the slepton mass splitting, which is possible to dominate over the contribution from the charged-lepton Yukawa couplings. A quantitative calculation of this threshold effect on $V_{\rm MNS}$ is strongly model-dependent and arbitrary [11], because it involves some unknown parameters of supersymmetric particles.

It is certainly a challenging task to examine how the relation $V_{\rm MNS} = V^\dagger_{\rm CKM} \mathcal{F}_\nu$ gets modified from the afore-listed radiative corrections, unless a realistic seesaw model is specifically given

and its free parameters are reasonably assumed. In this sense, we argue that the texture of $\mathcal{F}_\nu$ is unlikely to take a trivial form (such as the real bimaximal pattern) and the interesting relations in Eqs. (1) and (2) are more likely to be a numerical accident.

$\boxed{\mathbf{5}}$ In summary, we have investigated the natural correlation between the MNS lepton mixing matrix $V_{\rm MNS}$ and the CKM quark mixing matrix $V_{\rm CKM}$ for a class of seesaw models with grand unification. The correlation matrix $\mathcal{F}_\nu$ is found to be phenomenologically disfavored to take the bimaximal mixing form, no matter whether the nontrivial phases of $\mathcal{F}_\nu$ are taken into account or not. This observation turns out to be contrary to some previous arguments that the CKM matrix might measure the deviation of the MNS matrix from exact bimaximal mixing. We have also shown that a slight modification of the bimaximal mixing pattern of $\mathcal{F}_\nu$ may allow us to reproduce the quark-lepton complementarity relation, provided the relevant phase parameters get fine-tuned. Calculating $\mathcal{F}_\nu$ directly in terms of the mixing angles and CP-violating phases of $V_{\rm CKM}$ and $V_{\rm MNS}$, we have demonstrated a striking feature of $\mathcal{F}_\nu$: its (1,3) element cannot be vanishing or too small, even if the (1,3) elements of $V_{\rm MNS}$ and $V_{\rm CKM}$ are taken to be zero. It is therefore concluded that the texture of $\mathcal{F}_\nu$ is rather nontrivial in the seesaw models. We argue that this conclusion is in general not expected to be changed by possible quantum corrections and their threshold effects.

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FIGURES

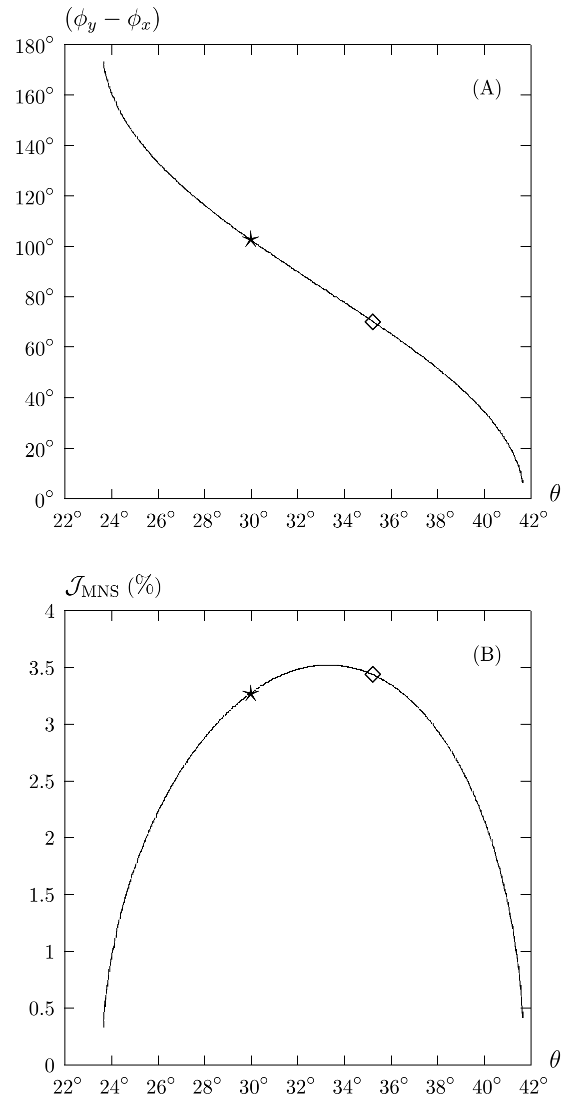

FIG. 1. (A) numerical illustration of the correlation between θ and $(\phi_y - \phi_x)$ for $\theta_{12} + \vartheta_{12} \approx 45^\circ$ to hold; (B) numerical dependence of $\mathcal{J}_{\rm MNS}$ on θ. Here $\vartheta_{12} \approx 13^\circ$ has been input. The *star* and *diamond* points stand for the neutrino mixing ansatz proposed in Ref. [21] with $\theta = 30^\circ$ and that in Ref. [22] with $\theta \approx 35.3^\circ$, respectively.